# Revision answers: Algebra, functions and equations (Topics 1 & 2)

**Coursebook chapters: 1–8; 15; 25**

**1.** (a) ℝ

(b) *f* −1(*x*) = e2*x*

(c) *f* −1(5) = e10 *[5 marks]*

**2.** *a* – *b*i + 2*a* + 2*b*i = 3i ⇒ *a* = 0, *b* = 3 *[4 marks]*

**3.** 8*a* – *b* = −13, *[5 marks]*

**4.** (a) *y* > 1

(b) 3(4e*x* + 1)2 = 75 ⇒ 4e*x* + 1 = 5 ⇒ x = 0 *[8 marks]*

**5.** (a)  = 40

(b) 8! – 2 × 7! = 30240 *[6 marks]*

**6.** (a) Use GDC to sketch *y* = 2(*x* − 1)2 − 6

(b) From GDC: −1.35 < *x* < 3.35 *[5 marks]*

**7.** (a) Arithmetic series, *u*1 = 500, *d* = 25, *S*20 = 14750

(b) Geometric, *u*1 = 500, *r* = 1.05, *un* = 500 × 1.05*n* − 1 > 1000 ⇒ *n* = 16 (from GDC)

(c) (1000 + 25(*n* – 1)) = 5000 gives *n* = 27 days *[9 marks]*

**8.** (a) *x* =

(b) *f*(*x*) = *f* −1 is equivalent to *f*(*x*) = *x* (because the two graphs cross on the line *y* = *x.*

 = *x* ⇒ *x* = −0.436 or 3.44 *[5 marks]*

**9.** (a) *p*(3) = 0

(b) *p*(*x*) = (*x* – 3)(2*x*2 + *x* – 3) = (*x* – 3)(2*x* + 3)(*x* – 1)

(c) Cubic graph with *x*-intercepts −1.5, 1, 3 and *y*-intercept 9. *[9 marks]*

**10.** The inductive step is:

For *n* = *k*, 5*k* +9*k* + 2 = 4*A*, *n* = *k*,

so for *n* = *k +* 1, 5*k* + 1 + 9*k* + 1 + 2 = 5 × 5*k* + 9 × (4*A* – 5*k* – 2) + 2 = 36*A* – 4 × 5*k* − 16

= 4(9*A* – 5*k* – 4) divisible by 4

*[9 marks]*

**11.** Gaussian elimination gives:



So, *z* = *t*, *y* = 0.2, *x* = 5.4 − *t* *[7 marks]*

**12.** (a) The inductive step uses the compound-angle formulae for sine and cosine.   
(See Worked example 15.19 for a similar method.)

(b) (i) *ω* =

(ii) 1 + *ω* + *ω*2 + *ω*3 + *ω*4 =  = 0 as *ω*5 = 1 *[10 marks]*

**13.** *x*1 + *x*2 = −, *x*1 − *x*2 = 1 ⇒ (*x*2 + 1) + *x*2 = −



 ⇒ *b*2 – *a*2 = 4*ac [6 marks]*

**14.** *xn* + *npxn* − 1 +*p*2*xn* – 2 = *xn* + 20*xn* – 1 + 180*xn* − 2

⇒ *np* = 20, *p*2 = 180

 = 180 ⇒ *n* = 10, *p* = 2 *[6 marks]*

**15.** *p*(*x*) = (*x*2 + 3*x* + 2)*q*(*x*) + (5*x* + 1)

*p*(−2) = (0)*q*(*x*) + (−10 + 1) = −9 *[4 marks]*

**16.** (a) *α* + *β* + *γ* = −*b* and *γ = α* + *β* ⇒ 2(*α* + *β*) = −*b*

*αβγ* = −*d* ⇒ *αβ* = −

(b) If *α* and *β* are solutions of the quadratic equation then *x*2 + *mx* + *n* = (*x* – *α*)(*x* – *β*).

Therefore *α β* = *n* and –(*α + β*) = *m*, which gives:

*m* =, *n* = 

(c) Three real roots when *α*, *β* are real, i.e. *m*2 – 4*n* ≥ 0 ⇔  ⇔ *b*3 ≥ 32*d* (as *b* > 0). *[9 marks]*